A Note on the Cosmic Evolution of the Axion in a Strong Magnetic Field

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It has been pointed out in the literature that in the presence of an external magnetic field the axion mass receives an electromagnetic contribution. We show that if a magnetic field with energy density larger than $\sim 10^{-8}$ times the energy density of the Universe existed at temperatures of a few GeV, that contribution would be dominant and consequently the cosmic evolution of the axion field would change substantially. In particular, the expected axion relic abundance would be lowered, allowing a small relaxation of the present cosmological bound on the Peccei-Quinn constant.

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The existence of the axion field is predicted by the Peccei-Quinn (PQ) mechanism [2] for the solution of the strong CP problem, one of the most puzzling points of modern particle physics (for a review see, e.g., [3]). This problem is related to the presence of the P- and CP-violating term $\mathcal{L}_\Theta = (\alpha_s/8\pi)\Theta\, G\tilde{G}$, known as $\Theta-$term, in the QCD Lagrangian. Here, α_s is the fine structure constant of the strong interactions, while G and $\tilde{G}$ are the gluon field and its dual. In the PQ-mechanism the parameter Θ becomes a dynamical field, the axion itself $a = \Theta f_a$, which emerges as the (pseudo-)Goldstone mode of the PQ-symmetry $U(1)_{\rm PQ}$, spontaneously broken at the energy scale f_a. The parameter f_a, known as the PQ- or axion-constant, characterizes the all axion phenomenology [4], and is presently constrained in the very narrow region $10^9 \lesssim f_a \lesssim 10^{12}$GeV by astrophysical and cosmological considerations [5]. The axion potential, generated by the non trivial axion-gluon interactions, is minimized for the CP-even configuration $\langle a \rangle = 0$, providing therefore a dynamical explanation for the CP-conserving behavior of the strong interactions.

The cosmic evolution of the axion field is described by the equation [6]

$$\ddot{\Theta} + 3H\dot{\Theta} + m_a^2(T)\Theta = 0, \tag{1}$$

where $H \simeq 1.66 g_*^{1/2} T^2/m_{\rm Pl}$ is the Hubble parameter (in the radiation era) with g_* the total number of effectively massless degrees of freedom and $m_{\rm Pl}$ the Planck mass. The temperature-dependent axion mass is [7] (see also [6])

$$m_a(T) \simeq \begin{cases} 0.1m\,(\Lambda/T)^{3.7} & (T \gg \Lambda), \\ m & (T \ll \Lambda), \end{cases} \tag{2}$$

where m is the zero temperature limit $m \simeq 6.2 \times 10^{-6}{\rm eV}/f_{12}$, $f_{12} = f_a/(10^{12}{\rm GeV})$, and $\Lambda \sim 200$MeV the QCD scale.

For temperatures high enough for the mass term in Eq. (1) to be negligible, the axion has no dynamics, and therefore Θ remains fixed on its initial value Θ_i, which is not required to be zero. However, as the axion mass becomes dominant on the friction (Hubble) term, Θ begins to oscillate with the frequency $m_a(T)$ and will eventually approach the CP-conserving limit $\Theta_{\rm today} \sim 0$. During this period of coherent oscillations the number of axions in a comoving volume remains constant and therefore the axion relic abundance today can be straightforwardly evaluated as

$$\Omega_a \simeq 1.6\,\Theta_i^2\, g_{*1}^{-1/2} f_{12}\, \frac{\rm GeV}{T_1}\,, \tag{3}$$

where the temperature T_1, defined by the equation $m_a(T_1) = 3H(T_1)$, indicates approximately the time when the oscillations start, and $g_{*1} = g_*(T_1)$.

If the only contribution to the axion mass were given by the QCD instanton effects (2), then $T_1 \simeq 0.9\,\Lambda_{200}^{0.65} f_{12}^{-0.175}$ GeV, where $\Lambda_{200} = \Lambda/200$MeV. In this approximation Eq. (3) reduces to $\Omega_a \simeq 0.2\,\Lambda_{200}^{-0.65}\Theta_i^2 f_{12}^{1.175}$. With the natural choice $\Theta_i \simeq 1$ [6], this gives $\Omega_a \simeq 0.3$ (the expected dark matter abundance) for $f_{12} \simeq 1$. Much larger values of f_{12} would cause too much axion production and are therefore excluded. This observation leads to the upper limit on the PQ-constant mentioned above, the so called cosmological bound $f_a \lesssim 10^{12}$GeV [8].

However, as pointed out in Ref. [1], in an external uniform magnetic field $B \gg B_c \simeq 4.4 \times 10^{13}$G ($B_c$ is the critical or Schwinger value) the axion mass receives an electromagnetic contribution, [1]

$$\delta m_a(B) \simeq 5.8\,\xi \left(\frac{B}{10^{23}\mathrm{G}}\right)^{1/2} \frac{10^6\mathrm{GeV}}{f_a}\,\mathrm{eV}, \tag{4}$$

where ξ is a model-dependent parameter of order of unity related to the effective axion photon coupling $g_{a\gamma} = \xi\alpha_{\rm em}/(2\pi f_a)$. (It is worth noting that the total axion mass is given by $m_{\rm tot}^2 = m_a^2 + \delta m_a^2$.) As we will show below this result has important consequences for axion cosmology. Indeed, if a magnetic field with energy density larger than about 10^{-8} times the energy density of the Universe existed before the period of the axion coherent oscillations, the electromagnetic contribution to the axion mass would be dominant, and the beginning of the oscillations would consequently start earlier. The most important consequence of this result is a reduction of the expected axion relic abundance [see Eq. (3)], and therefore a relaxation of the cosmological upper bound on the PQ-constant.

The existence of very intense magnetic fields in the early Universe is not excluded [10, 11]. Indeed, it has been invoked for the explanation of the presently observed large-scale magnetic fields, and could have interesting repercussions on the axion phenomenology (see, e.g., [12]). Since the primordial plasma is an excellent conductor, magnetic fields are frozen into the plasma and evolve as $B \propto T^2$. Introducing the constant b as $B = bT^2$, we can parameterize the evolution of the magnetic field as $B \simeq 1.4 \times 10^{19} b\,(T/\mathrm{GeV})^2$ G. Requiring the magnetic energy density $\rho_B = B^2/2$ to be less than the energy density of the Universe in the radiation era $\rho = (\pi^2/30)g_* T^4$ (to be consistent with the constraint on primordial magnetic fields coming from the Big Bang Nucleosynthesis and the Cosmic Microwave Background [11]), we find the maximum allowed value of b, $b_{\rm max} \simeq 0.8\, g_*^{1/2}$.

It is useful to re-write Eq. (4) as

$$\delta m_a(T) \simeq 0.2 \times 10^{-2} \xi\, b^{1/2} \Lambda_{200}\, m\, \frac{T}{\Lambda}\,, \tag{5}$$

and to introduce the temperature T_* such that the QCD axion mass (2) and the electromagnetic contribution (5) are equal, $\delta m_a(T_*) = m_a(T_*)$. It results in $T_* \simeq 2.2\,\xi^{-0.2} b^{-0.1} \Lambda_{200}^{-0.2} \Lambda$. Let us continue to represent the temperature at which the axion field begins to oscillate as T_1.
For $b \lesssim b_* = 0.9 \times 10^{-3}\,\xi^{-2} \Lambda_{200}^{1.5}\, f_{12}^{1.75}$ we have $T_1 < T_*$, thus the electromagnetic contribution to the axion mass is negligible with respect to that of QCD [see Eqs. (2) and (5)]. Therefore the standard analysis applies.
However, if $b \gtrsim b_*$ (which for $\Lambda_{200} = \xi = f_{12} = 1$ corresponds to a magnetic energy $\rho_B \gtrsim 10^{-8}\rho$) the electromagnetic contribution dominates and then T_1 is determined by imposing that $\delta m_a(T_1) = 3H(T_1)$. In this case we find $T_1 \simeq 0.2 \times 10^3\,\xi\, b^{1/2} f_{12}^{-1} g_{*1}^{-1/2}$ GeV. Then, inserting the value of T_1 in Eq. (3) we get

$$\Omega_a \simeq 0.9 \times 10^{-2} \Theta_i^2\, \xi^{-1} b^{-1/2} f_{12}^2\,. \tag{6}$$

Requiring $\Omega_a \lesssim 0.3$, we get that the maximum value of the Peccei-Quinn constant is $f_{12} \simeq 5.8\,\xi^{1/2} b^{1/4} \Theta_i^{-1}$ to which corresponds the temperature $T_1 \simeq 29.6\,\xi^{1/2} b^{1/4} \Theta_i\, g_{*1}^{-1/2}$ GeV. Because ξ and Θ_i are of order of unity, taking $b = b_{\rm max}$ we get $f_{12} \simeq 9.6$ and $T_1 \simeq 5.3$ GeV.

In conclusion, we have shown that a strong cosmological magnetic field can have a non-negligible influence on axion cosmology. In particular, the cosmological limit on the axion constant could be relaxed by one order of magnitude. In this case, the axion interactions with matter and photons would be reduced, rendering the axion more "invisible". As a final observation we note that the electromagnetic contribution to the axion mass, Eq. (4), was computed in the zero temperature limit and so our conclusions must be considered only as a preliminary result. Indeed, a careful calculation of the mass shift at finite temperature is in progress. Whatever the case, the phenomenon of axion mass shift in a strong magnetic field discussed in Ref. [1] needs to be considered seriously since it seems to be the most relevant effects of a uniform magnetic field on axion cosmology.

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[1] A uniform magnetic field, besides giving a contribution to the axion mass, causes a dissipation of the axion field itself. This is induced by the axion-photon conversion in the magnetic field. However, this affects only negligibly the expected axion relic abundance [9].

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